# A Review on Integration of Ultrasonic Vibration in Machining of Difficult-to-Cut Materials

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Abstract— This review compiles and analyzes forty-five recent studies exploring ultrasonic-assisted turning, milling, drilling, grinding, and hybrid techniques across diverse industries, including aerospace, automotive, biomedical, and energy. Each study is evaluated based on material type, machine setup, input parameters, and key output responses such as tool wear, surface integrity, delamination, and cutting temperature. The review highlights substantial improvements in machinability metrics but also reveals critical limitations, including scalability issues, lack of adaptive control, limited economic assessments, and insufficient long-term tool wear analysis. The paper concludes with research gaps in real-time adaptive control systems in ultrasonic vibrationassisted machining and proposes future directions for integrating intelligent control systems, AI-driven optimization, sustainable UVAM solutions compatible with Industry 4.0 environments.

*Index Terms*— Ultrasonic machining, titanium alloys, composite machining, vibration-assisted cutting, hybrid processes, optimization.

#### I. Introduction

The pursuit of advanced manufacturing capabilities has become increasingly critical in industries such as aerospace, biomedical, energy, and automotive engineering. These sectors demand materials that can perform reliably under extreme mechanical, thermal, and chemical conditions. Consequently, there has been a significant shift towards the use of high-performance materials such as titanium alloys (e.g., Ti-6Al-4V), nickel-based superalloys (e.g., Inconel 718), advanced ceramics (e.g., silicon carbide [SiC], aluminum oxide [Al<sub>2</sub>O<sub>3</sub>]), and fibre-reinforced composites (e.g., carbon fibre-reinforced polymer [CFRP], glass fibre-reinforced polymer [GFRP]). These materials are highly valued for their exceptional properties, including high strength-to-weight ratios, superior

corrosion resistance, excellent wear resistance, and stability at elevated temperatures. However, despite these advantages, they present significant challenges in terms of machinability. These difficult-to-cut materials typically exhibit low thermal conductivity, high abrasiveness, and a tendency to induce work hardening, which collectively contribute to rapid tool wear, excessive cutting forces, poor surface finish, and diminished dimensional accuracy when subjected to conventional machining techniques. Traditional approaches like high-speed machining or coolant optimization often fall short in addressing these issues comprehensively, especially in high-precision manufacturing environments.

To mitigate these machining difficulties, Ultrasonic Vibration-Assisted Machining (UVAM) has emerged as a cutting-edge technique. UVAM works by superimposing high-frequency vibrations—generally in the range of 20–40 kHz—onto the cutting tool or the workpiece. This modification results in intermittent tool-material contact, leading to a reduction in cutting forces, improved chip breakability, decreased heat generation, and enhanced tool life. In addition, UVAM often contributes to improved surface integrity and dimensional accuracy of the machined parts, making it particularly beneficial for applications requiring stringent tolerances and superior surface quality. Over the past decade, several variants of UVAM have been developed, including Ultrasonic-Assisted Turning (UAT), Ultrasonic-Assisted Milling (UAM), Ultrasonic-Assisted Drilling (UAD), and Ultrasonic-Assisted Grinding (UAG). These methods have been implemented across various materials and industrial settings with promising results. Despite the technological progress, the adoption of UVAM remains uneven, often limited to laboratory-scale experiments or pilot trials, with challenges related to system integration, scalability, and cost still under investigation.

This review aims to systematically evaluate the integration of UVAM techniques. The paper categorizes and analyzes peer-reviewed studies based on material types, machining processes,



ultrasonic configurations, industrial applications, and performance metrics. Key input parameters such as tool geometry, vibration amplitude, frequency, and feed rate are correlated with output parameters including tool wear, surface finish, chip morphology, and machining forces.

### II. LITERATURE REVIEW

The review of each entry summarizes the experimental setup, material studied, machine and technique used (e.g., ultrasonic-assisted turning, milling, drilling, grinding), as well as key input and output parameters. The outcomes are critically analyzed to extract significant findings such as improvements in tool life, surface integrity, chip evacuation, and dimensional accuracy. Identified limitations such as system complexity, lack of scalability, or inadequate multi-pass performance are also noted to highlight future research opportunities.

**Table 1.** Comprehensive Literature Review on the Application of Ultrasonic Vibration-Assisted Machining (UVAM) Techniques Across Various Industrial Materials

S r. N	Aut hor Na	Indust ry and Mater	Techni que and	Input Param eters	Outpu t Para	Key Findi	Limita tion
0	me	ial	Machi ne	eters	meter s	ngs	
1	Li et al. [1]	Aeros pace (Grade 5 titaniu m) (Ti- 6Al- 4V)	Used Ultraso nic Assiste d Turnin g on Lathe	Vibrati on frequen cy, feed rate	Cuttin g force, surfac e rough ness	Cuttin g force reduce d by 25–40% and better surfac e finish.	Not studied for dry cutting.
2	Zha ng et al. [2]	Aeros pace (Grade 5 titaniu m) (Ti- 6Al- 4V)	Ultraso nic Assiste d Milling on Vertica l Milling Machin e	Tool RPM, amplitu de	Flank wear, chip morph ology	35% flank wear reduct ion and better chip evacu ation.	No cost/int egratio n discussi on.
3	Ku mar & Sing h [3]	Biome dical (Grade 5 titaniu m) (Ti-6Al-4V)	Ultraso nic Assiste d Drilling on Vertica l Drilling Machin e	Spindle speed, vibratio n	Chip shape, temper ature	Better chip morph ology and coolin g.	No multi- pass data.
4	Pate 1 et al. [4]	Autom otive (Grade 5 titaniu m)	Elliptic al Ultraso nic Assiste d	Amplit ude, tool geomet ry	Tool life, adhesi on	Increa sed tool life and reduce	No parame tric optimiz ation.

		(Ti-	Turnin			d	
		6Al-	g on			adhesi	
_	a 1	4V)	Lathe	-	G 0	on.	
5	Sah u & Mis hra [5]	Medic al (Grade 5 titaniu m) (Ti- 6Al- 4V)	Dry Ultraso nic Assiste d Milling on CNC Milling Machin	Dry vs. wet conditi on	Surfac e integri ty	Surfac e integri ty impro ved under dry condit ion.	Increas ed vibratio n noise.
			e			IOII.	
6	Wan g et al. [6]	Energ y (Incon el 718)	Ultraso nic Assiste d Turnin g on Lathe	Tool wear, spindle speed	Residu al stress, machi nabilit y	Reduc ed residu al stress and impro ved machi nabilit y.	No fatigue testing post- process
7	Shar ma et al. [7]	Aeros pace (Incon el 718)	Ultraso nic Vibrati on- Assiste d Machin ing with Cryoge nic Coolin g	Coolant , amplitu de	Tool life, burrs	60% longer tool life and reduce d burrs.	High system comple xity.
8	Das et al. [8]	Energ y (Incon el 718)	Ultraso nic Assiste d Milling on CNC Milling Machin e	Feed rate, vibratio n	Tool wear	Tool wear reduce d by 40%.	Only low feed rates tested.
9	Rao & Pilla i [9]	Aeros pace (Incon el 718)	Ultraso nic Assiste d Milling with Minim um Quantit y Lubrica tion on VMC	Lubrica tion, speed	Surfac e rough ness	Surfac e finish signifi cantly impro ved.	Not applica ble to dry machin ing.
1 0	Prak ash et al. [10]	Industrial (Inconel 718)	Hybrid Ultraso nic Vibrati on- Assiste d Machin ing Setup	Tool design, frequen cy	Chip segme ntation	Minim ized vibrati on and better chip segme ntatio n.	Cost- benefit analysi s missing
1	Che n et	Biome dical Ceram	Ultraso nic Assiste	Depth of cut,	Surfac e mode	Enabl ed ductile	Low MRR

	al. [11]	ics (Silico n carbid e) (SiC)	d Grindin g on Cylindr ical Grindin g Machin	vibratio n		-mode remov al.	efficien cy.
1 2	Liu et al. [12]	Aeros pace Ceram ics (Alum inum oxide) (Al <sub>2</sub> O <sub>3</sub>	Ultraso nic Assiste d Drilling on Precisi on Drill Machin e	Tool type, frequen cy	Crack count, tool life	Reduc ed radial cracks and extend ed tool life.	Tool wear not deeply analyse d.
1 3	Hua ng et al. [13]	Aeros pace Ceram ics (Silico n nitride ) (Si <sub>3</sub> N <sub>4</sub> )	Ultraso nic Assiste d Grindin g on Surface Grindin g Machin e	Speed, depth	Grindi ng force	Reduc ed grindi ng force and cracki ng.	Needs validati on for other ceramic s.
1 4	Red dy & Kul karn i [14]	Electr onics / Medic al Ceram ics (Alum ina)	Ultraso nic Assiste d Drilling on Bench Drill Machin e	Spindle speed, amplitu de	Finish, damag e	Enhan ced surfac e finish and less damag e.	Spindle speed not optimiz ed.
1 5	Yan g & Zho u [15]	Electr onics (SiC)	Ultraso nic Vibrati on Micro machin ing System	Micro machin ing parame ters	Dimen sional accura cy	Superi or precisi on and dimen sional contro 1.	Limited to micro- scale machin ing.
1 6	Ku mar et al. [16]	Aeros pace, (Carbo n Fiber Reinfo rced Polym er) (CFRP )	Ultraso nic Assiste d Drilling on CNC Drill Setup	Tool speed, frequen cy	Delam ination	45% delami nation reduct ion.	Long- term tool wear not evaluat ed.
7	Pate 1 & Josh i [17]	Autom otive, (Glass Fiber Reinfo rced Plastic ) (GFRP	Ultraso nic Assiste d Milling on Vertica l Milling Machin e	Amplit ude, feed rate	Thrust force	Lower thrust force and better chip evacu ation.	Layer thickne ss variatio n not studied.
1 8	Nair et al. [18]	Aeros pace,	Ultraso nic Assiste	Vibrati on speed	Fiber pull- out	Reduc ed fiber	High feed rate

	ı	(GEP P				11	
	P:	(CFRP	d Drilling on Portabl e Drilling Machin e			pull- out and matrix damag e.	trials missing
1 9	Bha ndar i et al. [19]	Aeros pace, (CFRP	Orbital Ultraso nic Assiste d Drilling System	Orbital angle, speed	Hole quality , temper ature	High hole qualit y and lower tempe rature.	Limited to flat parts.
2 0	Sing h et al. [20]	Autom otive, (CFRP	Multi- mode Ultraso nic Assiste d Drilling on CNC Machin e	Multi- mode setup	Burrs, smoot hness	Burrs reduce d and smoot her surfac e.	Comple x vibratio n control require d.
2 1	Sing h & Meh ta [21]	Metall urgy (Cobal t- based Alloys	Ultraso nic Assiste d Turnin g on Precisi on Lathe	Tool type, frequen cy	Machi nabilit y, notch	Enhan ced machi nabilit y and less notch sensiti vity.	Only orthogo nal cutting evaluat ed.
2 2	Tiw ari et al. [22]	Metall urgy (Nimo nic Alloy)	Ultraso nic- Assiste d Electric al Dischar ge Machin ing Setup	EDM pulse, vibratio n	Tool wear, craters	Tool wear and crater format ion reduce d.	No quantit ative results.
2 3	Gos wam i & Tha kur [23]	Biome dical, (Bone- like)	Ultraso nic Assiste d Drilling on Orthop edic Drill Machin e	Tool RPM, force	Finish, tool force	Low tool force and good surfac e finish.	No biocom patibilit y testing.
2 4	Zen g et al [24].	Toolin g (Super hard Alloys )	Ultraso nic Vibrati on- Assiste d Turnin g on CNC Lathe	Tool geomet ry, frequen cy	Therm al distort ion	Reduc ed therm al distort ion during machi ning.	Effect of tool geomet ry not explore d.
2 5	Bhat t et al. [25]	Sustai nable (Multi - Materi al)	Hybrid Ultraso nic Vibrati on- Assiste	Energy input, tool type	Energ y use, sustain ability	Better energy efficie ncy and sustai	Econo mic viabilit y not analyse d.

			d Machin ing on Intellig ent Workst ation			nabilit y.	
2 6	Alav i et al. [26]	Aeros pace, (Ti- 6Al- 4V)	Ultraso nic Assiste d Milling on CNC Lathe	Amplit ude, feed rate	Tool wear, chip form	Reduc ed tool wear by 30% and impro ved chip curlin g.	Tool chatter under high depth of cut.
2 7	Ku mar et al. [27]	Power (Incon el 625)	Ultraso nic Assiste d Drilling on Vertica l Machin ing Center	Tool rpm, coolant type	Drill quality , thrust force	Impro ved hole round ness and lower thrust.	Not studied for dry machin ing.
2 8	Lee & Kim [28]	Dental (Zirco nia Ceram ic)	Ultraso nic Assiste d Grindin g on Surface Grinder	Vibrati on directio n, pressur e	Surfac e cracks, remov al rate	Suppr essed surfac e cracks and mainta ined ductile regim e.	High tool wear not resolve d.
2 9	Sing h et al. [29]	Aeros pace (CFRP Comp osite)	Ultraso nic Assiste d Milling on Vertica l Milling Machin e	Layer angle, tool type	Delam ination , surfac e rough ness	Reduc ed delami nation and better layer transit ion.	Effect on comple x contour s not studied.
3 0	Che n & Zha o [30]	Aeros pace (SiC)	Ultraso nic- Assiste d Electric al Dischar ge Machin ing System	Pulse duratio n, ultraso nic frequen cy	MRR, tool wear	Impro ved MRR and lower wear.	Electrol yte heating not studied.
3 1	Man dal et al. [31]	Energ y (Ni- based Alloy)	Ultraso nic Assiste d Milling with Cryoge nic Coolin g on CNC	LN2 flow, amplitu de	Tool temper ature, flank wear	Reduc ed flank wear and tempe rature rise.	Setup comple xity and cost high.

			Machin				
3 2	Rah man et al. [32]	Aeros pace (CFRP Panel)	Orbital Ultraso nic Assiste d Drilling on Jig Bore	Orbit speed, rpm	Burr format ion, drill quality	Small er burrs and impro ved edge qualit y.	Tested only on small sample size.
3 3	Zho u et al. [33]	Aeros pace (Ti- 6Al- 4V)	Ultraso nic Assiste d Turnin g on Precisi on Lathe	Amplit ude, depth of cut	Surfac e residu al stress	Lower tensile residu al stress after machi ning.	Not validate d for varying tool geomet ries.
3 4	Ver ma & Tiw ari [34]	Autom otive (GFRP )	Ultraso nic Assiste d Drilling with MQL on CNC Drill Machin e	Oil mist rate, tool rpm	Fiber pull-out, temper ature	Minim ized therm al damag e and pull- out.	Not tested under dry conditi ons.
3 5	Wan g & Liu [35]	Aeros pace (Incon el 718)	Ultraso nic Assiste d Turnin g on CNC Lathe	Tool angle, vibratio n power	Vibrat ion suppre ssion, finish	Tool vibrati on minim ized and better finish.	Limited to cylindri cal geomet ries.
3 6	Park et al. [36]	Ceram ic Proces sing (Al <sub>2</sub> O <sub>3</sub>	Ultraso nic Assiste d Grindin g on Tool Room Grinder	Feed, amplitu de	Crack propag ation, finish	Lower subsur face cracks and impro ved surfac e finish.	Tool dressin g frequen cy high.
3 7	Kun du et al. [37]	Hybrid Comp osite Multi- Materi al	Ultraso nic Assiste d Drilling on CNC Drilling Station	Stackin g sequent ial, amplitu de	Hole round ness, burr height	Better dimen sional contro l, fewer burrs.	Not tested on large holes.
3 8	Chat terje e et al. [38]	Aeros pace (Nimo nic 90)	Ultraso nic Assiste d Milling on CNC Milling Machin e	Vibrati on amp., tool coating	Tool life, heat zone	Coate d tools perfor med better under UAM.	Not tested under high- speed regime.
3 9	Lu & Zha	Advan ced Ceram	Ultraso nic Assiste d	Vibrati on mode,	Remo val mode, finish	Transi tion to ductile -mode	Tool lifetime still short.

	nσ	ics	Grindin	tool		observ	
	ng [39]	(Si <sub>3</sub> N <sub>4</sub> )	g on	wear		ed.	
			Surface				
			Grinder				
4	Josh	Biome	Ultraso	Tool	Drill	Impro	No
0	i et al.	dical	nic Assiste	type,	quality	ved bioco	implant
	[40]	(Bone Simula	d	rpm	, temp.	mpati	fatigue test
	[10]	tion	Drilling			ble	done.
		Polym	on			surfac	
		er)	Medica			e	
			l Drill Machin			finish.	
			e				
4	Zha	Aeros	Ultraso	Step	Surfac	Reduc	Slotting
1	ng	pace /	nic-	over,	e	ed	limited
	et al.	Slot	Assiste	frequen	rough	rough	to
	[41]	Millin g (Ti–	d Slot Milling	cy	ness, tool	ness and	shallow depths.
		6Al-	on		vibrati	stable	ucpuis.
		4V)	CNC		on	slottin	
		·	Slotter			g.	
4 2	Meh ta &	Aeros	Ultraso nic	Feed	Delam	Enhan	Only 2-
2	ta & Ku	pace / Autom	nic Assiste	rate, amplitu	ination , fiber	ced delami	layer laminat
	mar	otive	d	de	breaka	nation	e
	[42]	(Carbo	Drilling		ge	resista	tested.
		n	on			nce.	
		Epoxy Lamin	Multi- axis				
		ate)	Drilling				
			Setup				
4	Gao	Aeros	Ultraso	Feed,	Tool	Reduc	Only
3	& Lin	pace /	nic Aggista	speed	load, finish	ed	turning
	[43]	Energ y	Assiste d		minsn	cuttin g load	assesse d.
	[]	(Haste	Milling			and	
		lloy	on			better	
		X)	CNC			finish.	
			Milling Machin				
			e				
4	Tan	Additi	Ultraso	RPM,	Surfac	Enhan	Long
4	g et	ve	nic-	amplitu	e o · · ·	ced	tool
	al. [44]	Manuf acturin	Assiste d End	de	finish, tool	finish on	length affects
	[44]	g	a Ena Milling		marks	on 3D-	stabilit
		(AlSi1	on			printe	y.
		0Mg)	Vertica			d	
			1 CNC			parts.	
4	Iyer	Autom	Mill Ultraso	Reinfor	Hole	Less	No
5	&	otive /	nic	cement	accura	burr	study
	Ran	MMCs	Assiste	%,	cy,	format	on tool
	a	(SiC-	d	amplitu	burrs	ion in	degrada
	[45]	Reinfo rced	Drilling	de		high SiC	tion.
		MMC)	on Multi-			SiC compo	
	i	1,11,10,	materia			sites.	
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			l Workpi				
			l Workpi ece				
			l Workpi				

### III. RESEARCH GAP

The Literature review shows the benefits of ultrasonic vibration-assisted machining (UVAM) techniques, such as reduced tool wear, enhanced surface finish, and improved material removal mechanisms. Some of the key research gaps remain unaddressed, as mentioned below.

1. Lack of Real-Time Adaptive Control Systems
Most reviewed studies applied fixed ultrasonic
parameters (e.g., amplitude, frequency) throughout the
process. There is minimal exploration of intelligent,
sensor-based feedback systems that can adaptively
tune these parameters based on real-time process
variables like tool wear, temperature, and vibration.

### 2. Insufficient Long-Term Tool Performance Analysis

While short-term performance metrics such as surface roughness and thrust force are well-studied, very few papers examine tool failure over extended production cycles, especially under industrial-scale high-speed and high-feed conditions.

Limited Industrial Integration and Scalability
Many experiments are conducted on basic setups
(lathe, VMC, drill press) and rarely incorporate
UVAM into 5-axis CNCs, robotic arms, or automated
production lines. This hinders broader adoption in
industrial environments.

## 4. Underexplored Hybrid Techniques and Process Optimization

Although some hybrid approaches (UVAM + MQL/Cryogenic Cooling) are explored, there is no unified framework that combines UVAM with simulation-driven optimization (e.g., FEM, DOE) or AI/ML for process control.

5. Material and Technique Specificity Research tends to be narrowly focused—either on one material (e.g., Ti-6Al-4V) or one method (e.g., UAT). Broader comparative studies involving multiple materials and UVAM techniques under unified testing protocols are rare.

The author did not find adequate literature focusing on the integration of real-time adaptive control systems in ultrasonic vibration-assisted machining (UVAM) across industrial applications. Most existing research utilizes static ultrasonic parameters without sensor-based feedback mechanisms. Studies addressing tool wear under prolonged production cycles or the scalability of UVAM on complex geometries and multimaterial structures are also limited.

### IV. CONCLUSION

This review critically examined forty-five peer-reviewed studies focusing on the integration of Ultrasonic Vibration-Assisted Machining (UVAM) techniques in processing difficult-to-cut materials such as titanium alloys, nickel-based superalloys, advanced ceramics, and fibre-reinforced composites. The techniques surveyed included Ultrasonic Assisted Turning (UAT), Drilling (UAD), Milling (UAM), and Grinding (UAG), as well as advanced hybrid setups incorporating cryogenic cooling, Minimum Quantity Lubrication (MQL), and AI-driven enhancements. Collectively, these studies demonstrated consistent improvements in tool life, cutting efficiency, surface quality, and thermal management across aerospace, biomedical, energy, and automotive sectors. Notably, Alavi et al. reported a 30% tool wear reduction when

milling Ti-6Al-4V, while Das et al. observed 40% less tool wear in Inconel 718 with UVAM. Liu et al. achieved crack suppression in Al<sub>2</sub>O<sub>3</sub> ceramics, and Bhandari et al. enhanced hole quality and reduced thermal load in CFRP via orbital UAD. Despite these gains, challenges persist—most notably the lack of real-time adaptive control, scalability to complex geometries, long-term performance data, and cost-benefit validation. Therefore, future research should focus on intelligent feedback-based systems, hybrid optimization using digital twins or Artificial Intelligence (AI)/Machine Learning (ML) tools, and quantifiable economic and environmental assessments to enable UVAM's full-scale industrial adoption in smart manufacturing ecosystems.

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